

GENERALIZED THEORY OF UNCERTAINTY (GTU) — PRINCIPAL CONCEPTS AND IDEAS¹

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Abstract

Uncertainty is an attribute of information. The path-breaking work of Shannon has led to a universal acceptance of the thesis that information is statistical in nature. Concomitantly, existing theories of uncertainty are based on probability theory. The generalized theory of uncertainty (GTU) departs from existing theories in essential ways. First, the thesis that information is statistical in nature is replaced by a much more general thesis that information is a generalized constraint, with statistical uncertainty being a special, albeit important case. Equating information to a generalized constraint is the fundamental thesis of GTU.

Second, bivalence is abandoned throughout GTU, and the foundation of GTU is shifted from bivalent logic to fuzzy logic. As a consequence, in GTU everything is or is allowed to be a matter of degree or, equivalently, fuzzy. Concomitantly, all variables are, or are allowed to be granular, with a granule being a clump of values drawn together by a generalized constraint.

And third, one of the principal objectives of GTU is achievement of NL-capability, that is, the capability to operate on information described in natural language. NL-capability has high importance because much of human knowledge, including knowledge about probabilities, is described in natural language. NL-capability is the focus of attention in the present paper.

The centerpiece of GTU is the concept of a generalized constraint. The concept of a generalized constraint is motivated by the fact that most real-world constraints are elastic rather than rigid, and have a complex structure even when simple in appearance. Briefly, if X is a variable taking values in a universe of discourse, U , then a generalized constraint on X , $GC(X)$, is an expression of the form $X \text{ isr } R$, where R is a constraining relation, and r is an indexical variable which defines the modality of the constraint, that is, its semantics. The principal constraints are possibilistic ($r=\text{blank}$); veristic ($r = v$); probabilistic ($r = p$); random set ($r = r$); fuzzy graph ($r=fg$); usuality ($r = u$); bimodal ($r=bm$); and group ($r = g$). Generalized constraints may be combined, qualified, propagated and counterpropagated. A generalized constraint may be a system of generalized constraints. The collection of all generalized constraints constitutes the generalized constraint language, GCL.

The fundamental theses of GTU may be expressed as the symbolic equality

$$I(X) = GC(X),$$

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